RUHR UNIVERSITÄT BOCHUM RUB

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Hausübungen zur Vorlesung

Quantenalgorithmen

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Exercise 1 (3 Punkte):
Calculate the tensor products $X \otimes Y, Y \otimes X, Y \otimes Y$ for

$$X = \begin{pmatrix} 1 & 0 & -1 \\ i\sqrt{2} & 2 & 0 \\ 1 & 0 & \sqrt{2} \end{pmatrix}, \quad Y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Exercise 2 (5 Punkte):
Let $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$. Prove that for $x \in \{0,1\}^n$

$$H \otimes H \otimes \ldots \otimes H|x\rangle = H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{\langle x,y \rangle} |y\rangle,$$

where $\langle x, y\rangle = x_1y_1 + x_2y_2 + \ldots x_ny_n$.

Exercise 3 (3 Punkte):
Given $|z_1\rangle = \begin{pmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{pmatrix}, |z_2\rangle = \begin{pmatrix} -\sin\theta \\ e^{-i\phi}\cos\theta \end{pmatrix}$, check that $\{|z_1\rangle, |z_2\rangle\}$ form an orthonormal basis in $\mathbb{C}^2$. Find an orthonormal basis for $\mathbb{C}^4$ by tensoring.

Exercise 4 (4 Punkte):
Consider the states:

$$\begin{aligned} |\beta_{00}\rangle &= \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ |\beta_{01}\rangle &= \tfrac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ |\beta_{10}\rangle &= \tfrac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\ |\beta_{11}\rangle &= \tfrac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \end{aligned}$$

1. Show that $\{|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle\}$ form a basis (knows as Bell basis) in $\mathbb{C}^4$;
2. Let $U \in \mathbb{C}^{2\times 2}$ be unitary. Show that for $|z\rangle = U|0\rangle = \alpha|0\rangle + \beta|1\rangle$ we obtain $|z^\perp\rangle = U|1\rangle = -\overline{\beta}|0\rangle + \overline{\alpha}|1\rangle$. We call $|z\rangle$, $|z^\perp\rangle$ a rotation of $|0\rangle$, $|1\rangle$.
3. Prove that Bell states $|\beta_{00}\rangle, |\beta_{11}\rangle$ are rotationally invariant, that is $(U \otimes U)|\beta_{ii}\rangle = |\beta_{ii}\rangle$ for $i \in \{0,1\}$ and any unitary $U \in \mathbb{C}^{2\times 2}$.