

RUHR-UNIVERSITÄT BOCHUM

Standard Security Does Imply Security Against Selective Opening for Markov Distributions

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HGI - Ruhr University Bochum, IST Austria

Selective Opening Attacks

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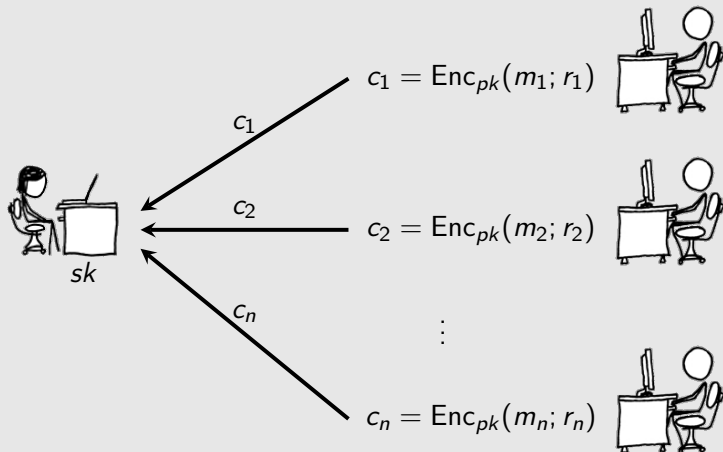


Image source: xkcd.com

Selective Opening Attacks

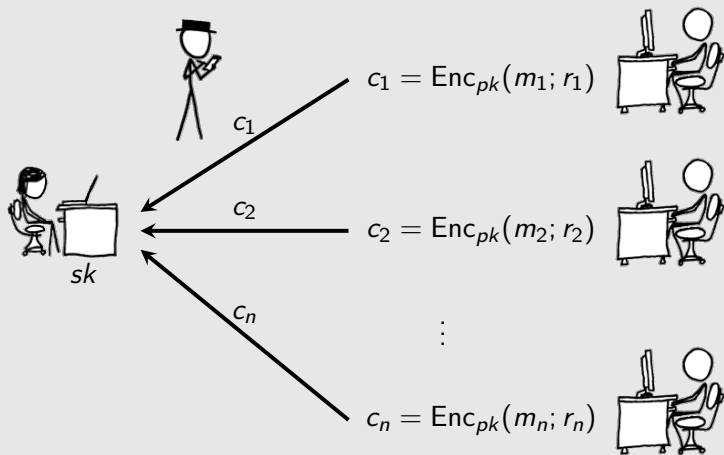


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Selective Opening Attacks

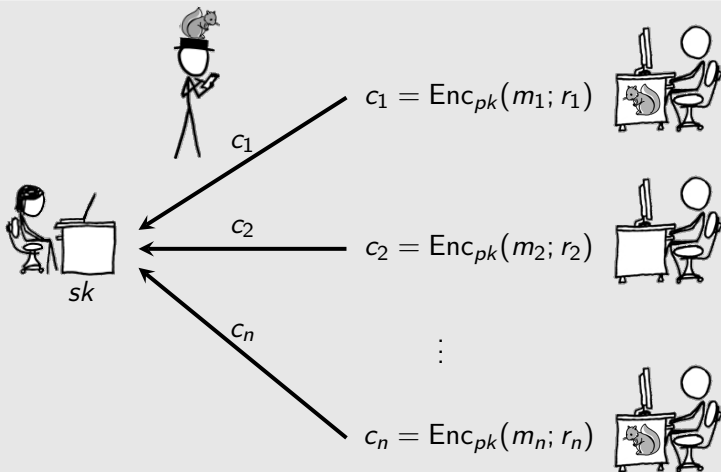


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Selective Opening Attacks

Do the messages of uncorrupted parties remain confidential?

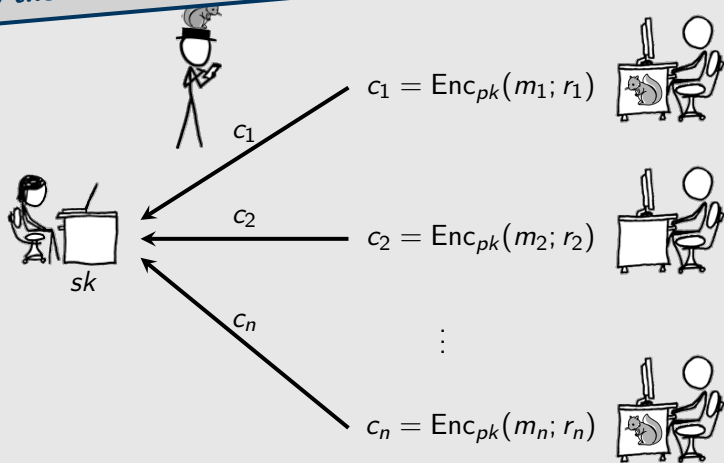
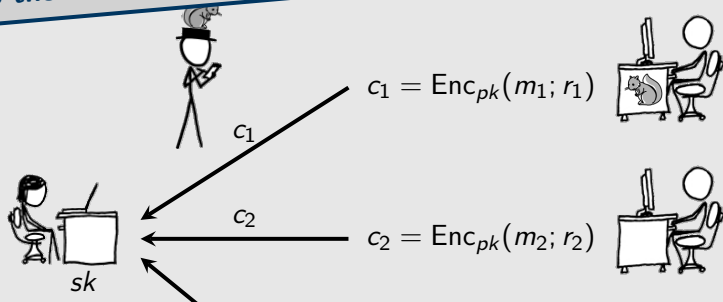


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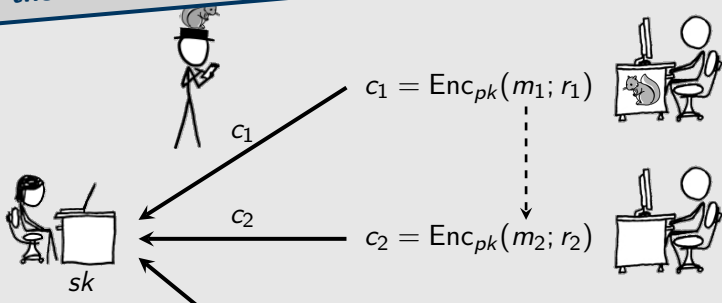
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- Naturally arises in secure MPC

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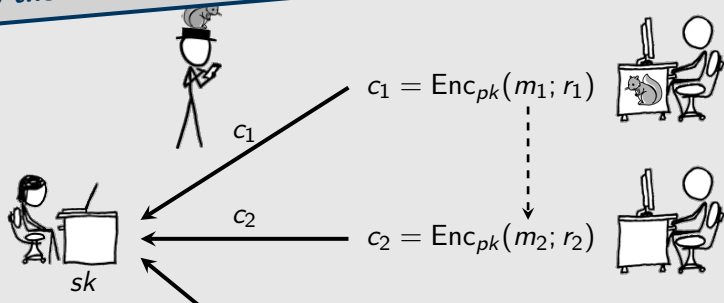


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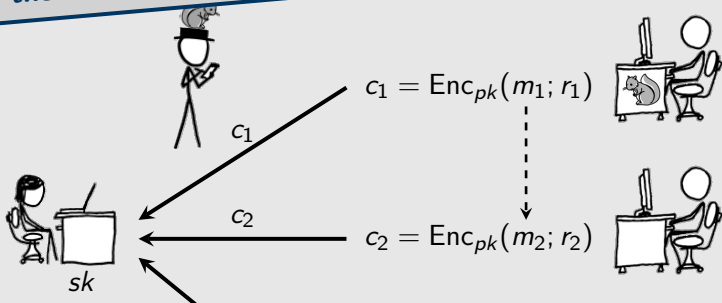


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- **Not** implied by (standard) IND-CPA [HRW15]

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- Naturally arises in secure MPC
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- **Not** implied by (standard) IND-CPA [HRW15]
- Dates back to [DNRS99]

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Defining IND-SO-CPA Security

Adversary obtains pk and submits a distribution \mathcal{D} of its choice.

$$(m_1, \dots, m_n) \leftarrow \mathcal{D}$$

$$\boxed{m_i} := Enc_{pk}(m_i; r_i)$$

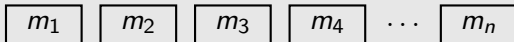


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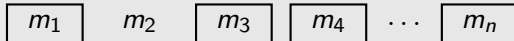


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where m_3, m_4, \dots are **resampled** conditioned on every opened ciphertext.



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$(m_1$	m_2	m_3	m_4	\dots	$m_n)$
\approx_c					
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Defining IND-SO-CPA Security

IND-SO-CPA_b

 $(pk, sk) \leftarrow \text{Gen}$
 $(m_1, \dots, m_n) \leftarrow \mathcal{D}$
 $r_1, \dots, r_n \leftarrow \text{Coins}$
 $c_i := \text{Enc}_{pk}(m_i; r_i)$

Resampling:

 $(m_1, \dots, m_n) \leftarrow \mathcal{D}, \text{ s.t.}$
 $m_i = m_i \text{ for all } i \in \mathcal{I}$
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 \mathcal{D}
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 $b = 0 : (m_1, \dots, m_n)$
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A

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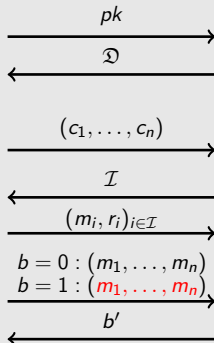
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PKE is IND-SO-CPA secure if for all ppt \mathcal{A} and all eff. resamp. \mathcal{D} :

$$\left| \Pr[\text{IND-SO-CPA}_0^{\mathcal{A}} \Rightarrow 1] - \Pr[\text{IND-SO-CPA}_1^{\mathcal{A}} \Rightarrow 1] \right| \leq \text{negl.}$$

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Naïve reduction loses 2^n to know m_1, \dots, m_n in advance.

IND-SO-CPA is IND-SO-CPA secure if for all ppt \mathcal{A} and all eff. resamp. \mathcal{D} :

$$\left| \Pr[\text{IND-SO-CPA}_0^{\mathcal{A}} \Rightarrow 1] - \Pr[\text{IND-SO-CPA}_1^{\mathcal{A}} \Rightarrow 1] \right| \leq \text{negl.}$$

When does IND-CPA imply IND-SO-CPA?

Theorem 1

Let PKE be any IND-CPA secure scheme. Then PKE is IND-SO-CPA secure w.r.t. to Markov distributions.

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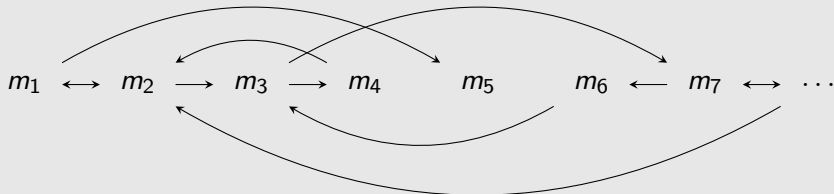
Theorem 1

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- First positive result for non-trivial distributions
- Loss of n^3 (see paper)
- Result extends to *low-dependency* distributions (see paper)

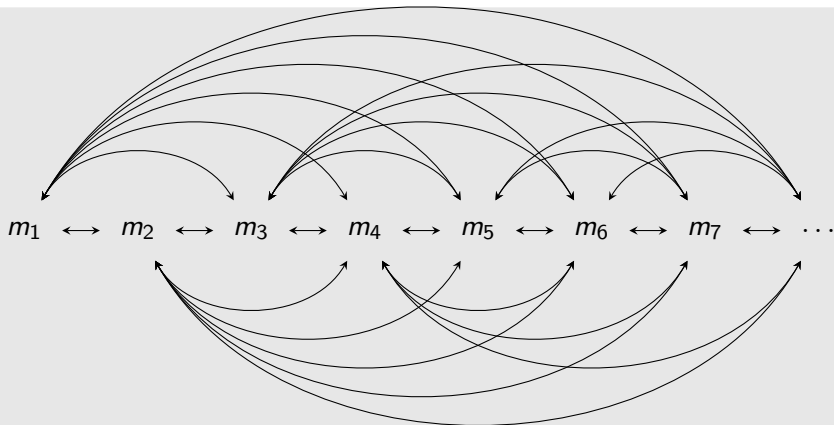
ePrint: 2015/853

When does IND-CPA imply IND-SO-CPA?



Distribution of a message m_i is a function of its immediate predecessors only.

When does IND-CPA imply IND-SO-CPA?



Wishful thinking

When does IND-CPA imply IND-SO-CPA?

m_1 m_2 m_3 m_4 m_5 m_6 m_7 ...

State of the Art

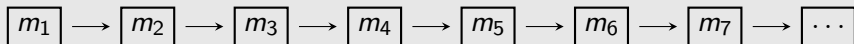
[DNRS99,BY09]

When does IND-CPA imply IND-SO-CPA?

$$m_1 \longrightarrow m_2 \longrightarrow m_3 \longrightarrow m_4 \longrightarrow m_5 \longrightarrow m_6 \longrightarrow m_7 \longrightarrow \dots$$

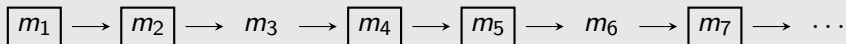


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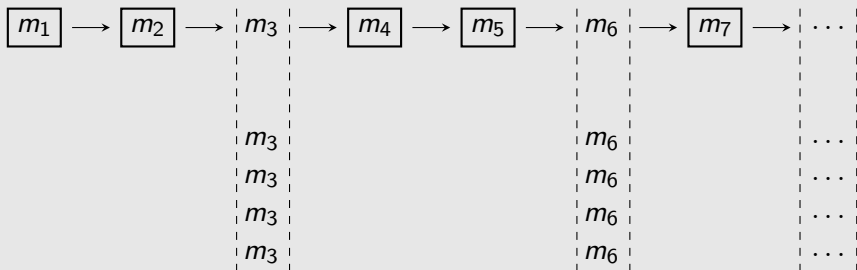
When does IND-CPA imply IND-SO-CPA?

Assume that \mathcal{A} opens m_3 , m_6 and m_8, \dots, m_n



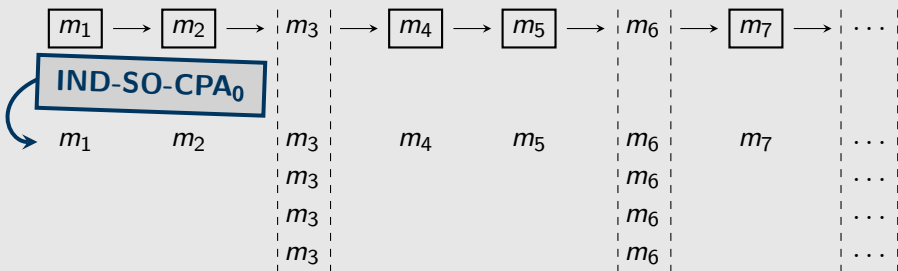
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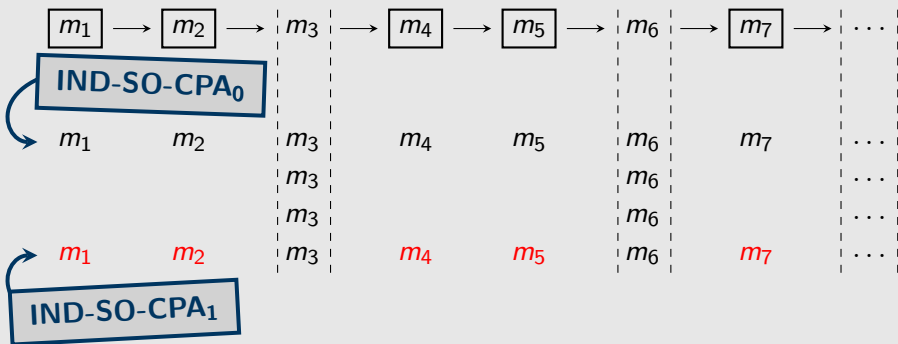
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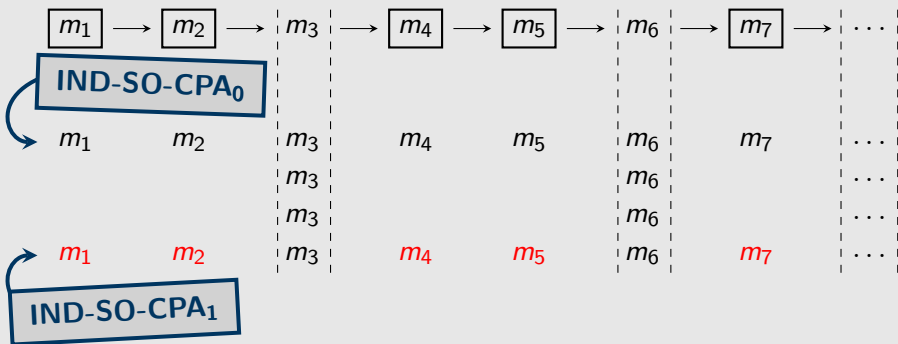
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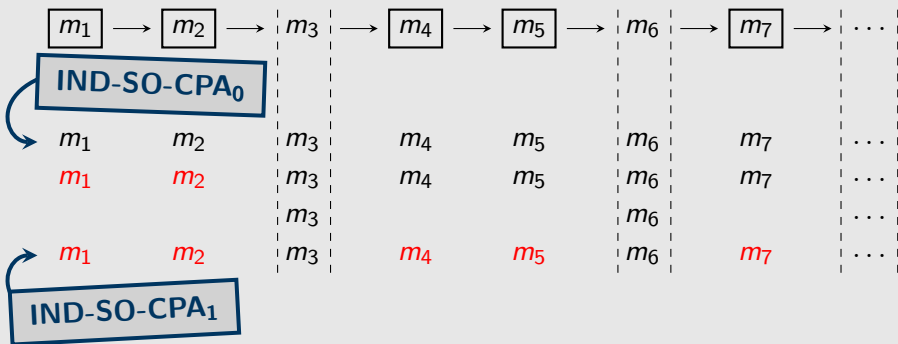
Hybrid i : Return **resampled** messages up to the i^{th} opening query.



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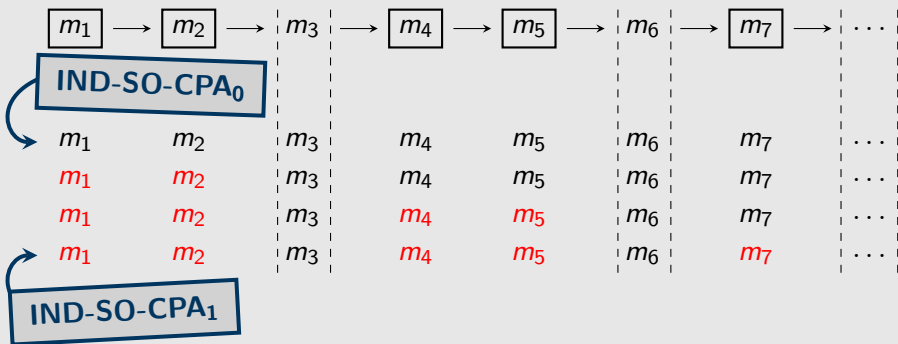
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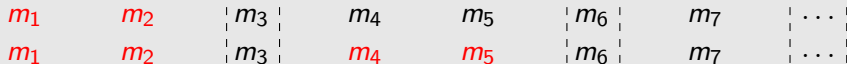
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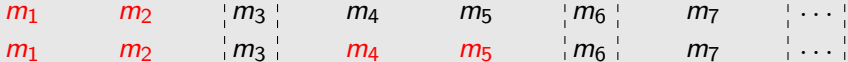


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Guess interval $[m_4, m_5]$ confined by consecutive opening queries.



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Resample $(m_1, \dots, m_n) \leftarrow \mathfrak{D}$ conditioned on $m_3 = m_3$ and $m_6 = m_6$.



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Submit $[m_4, m_5]$ and $[m_4, m_5]$ to (vectorial) IND-CPA challenger.

m_1	m_2	m_3	m_4	m_5	m_6	m_7	...
m_1	m_2	m_3	m_4	m_5	m_6	m_7	...

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Reduction encrypts m_1, \dots, m_3 and m_6, \dots, m_n on its own.

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Open queries might happen on the left of m_3 .

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Resample $(\tilde{m}_1, \dots, \tilde{m}_n) \leftarrow \mathcal{D}$ conditioned on every opened ciphertext.

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Open queries might happen on the left of m_3 .

Resample $(\tilde{m}_1, \dots, \tilde{m}_n) \leftarrow \mathcal{D}$ conditioned on every opened ciphertext.

Use m_3 to glue **resampled** and sampled partial vectors together.

m_1	m_2	m_3	m_4	m_5	m_6	m_7	...
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Thank you!

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